Estimation of Standard Auction Models

EC 2022

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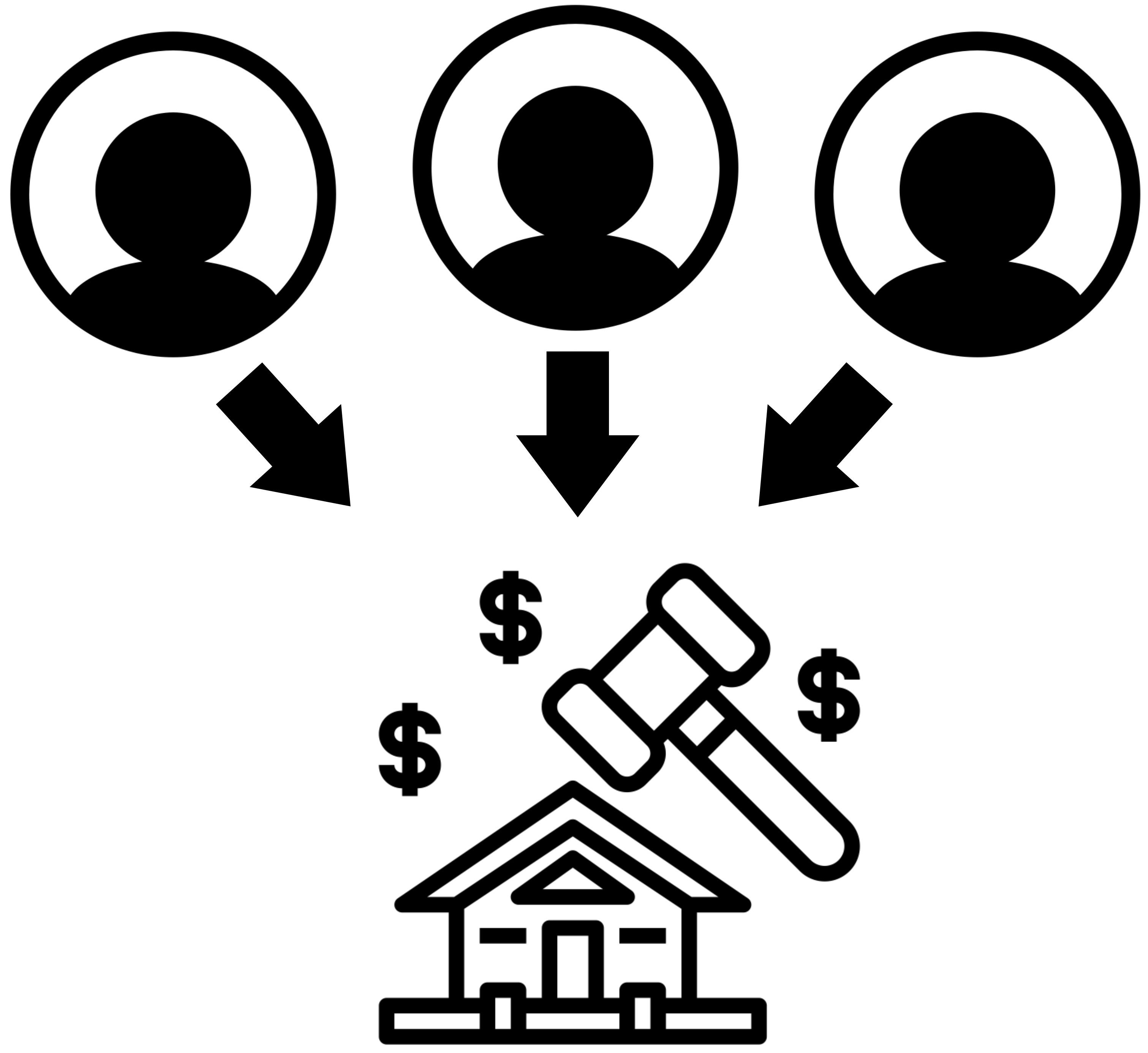

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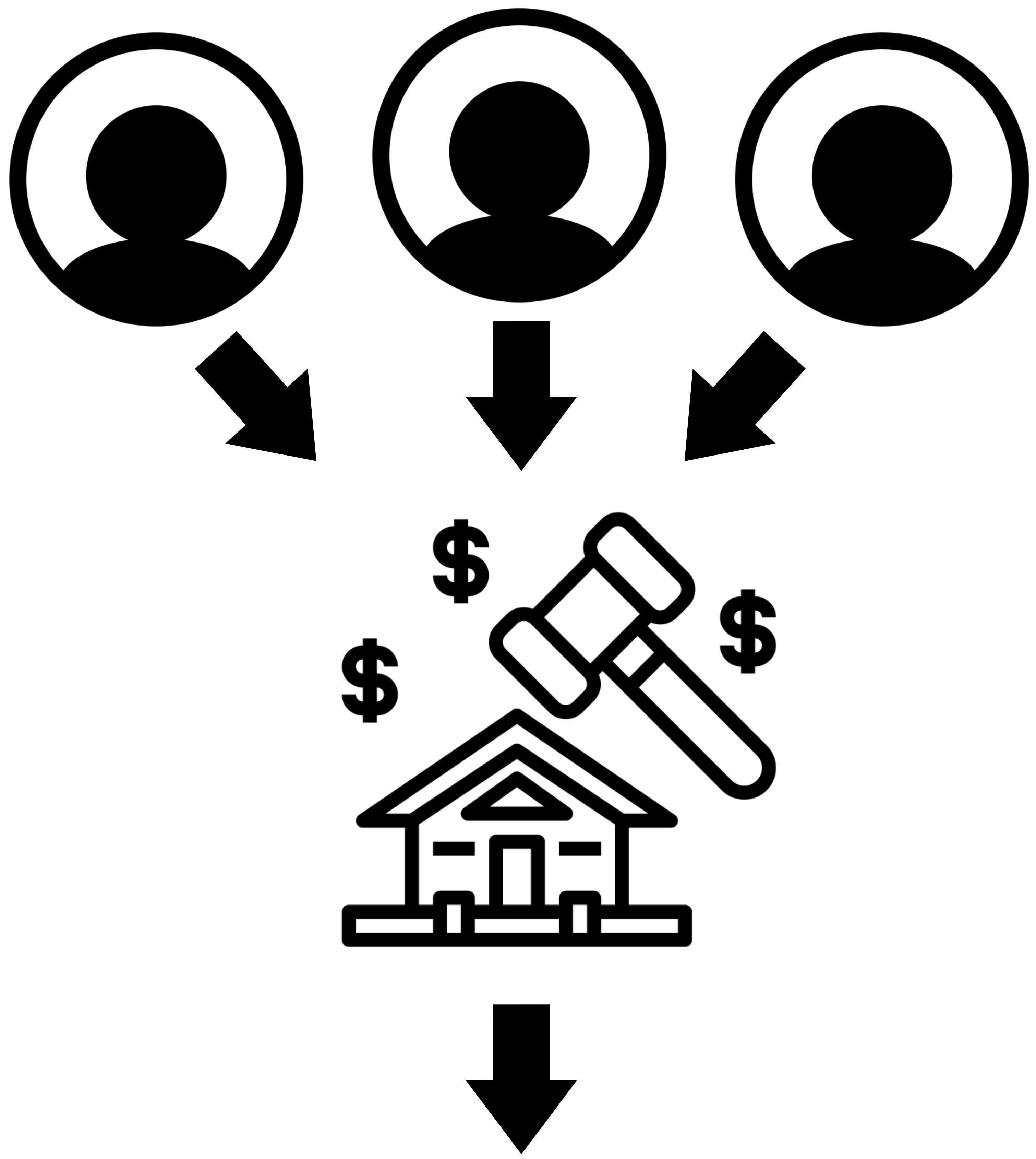
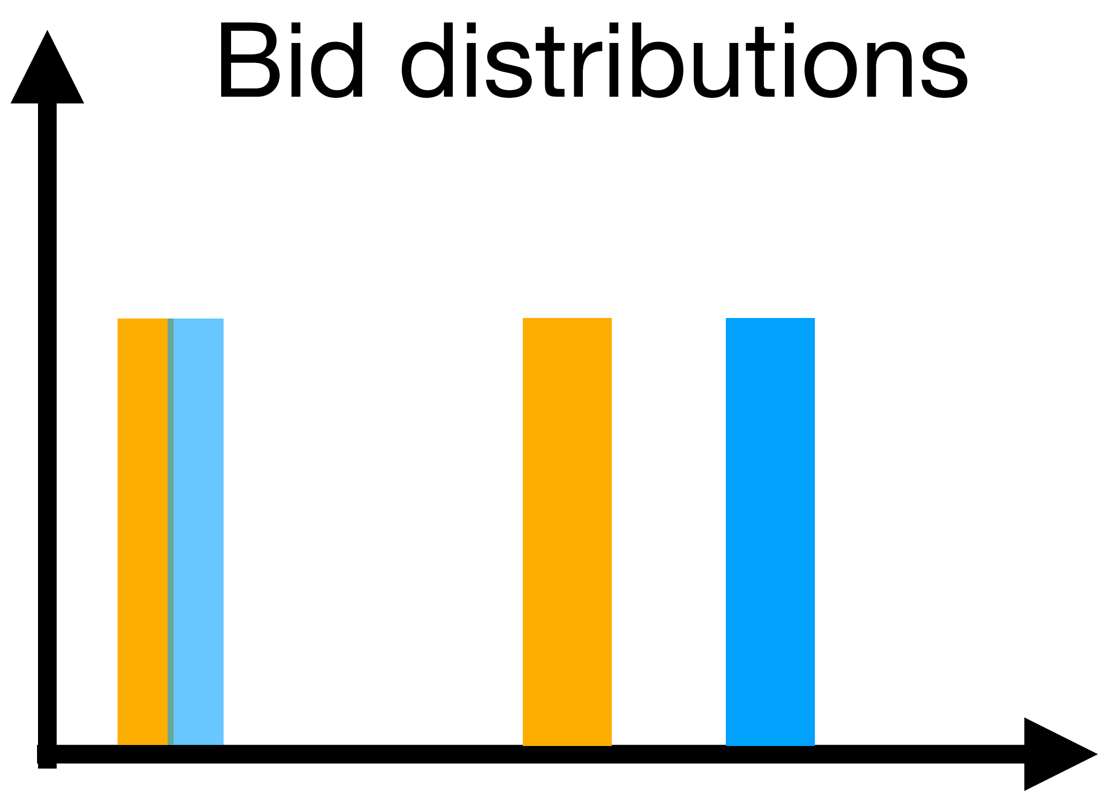
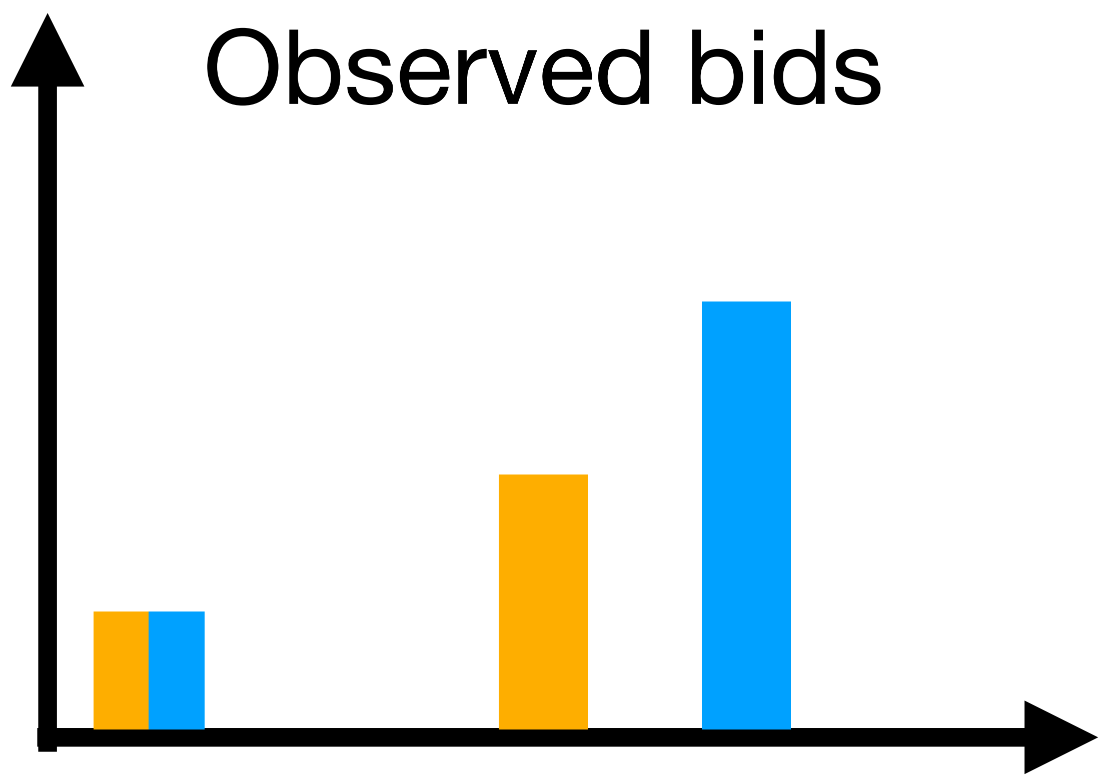


"Agent W won and paid \$$Y$"

Technical Challenges





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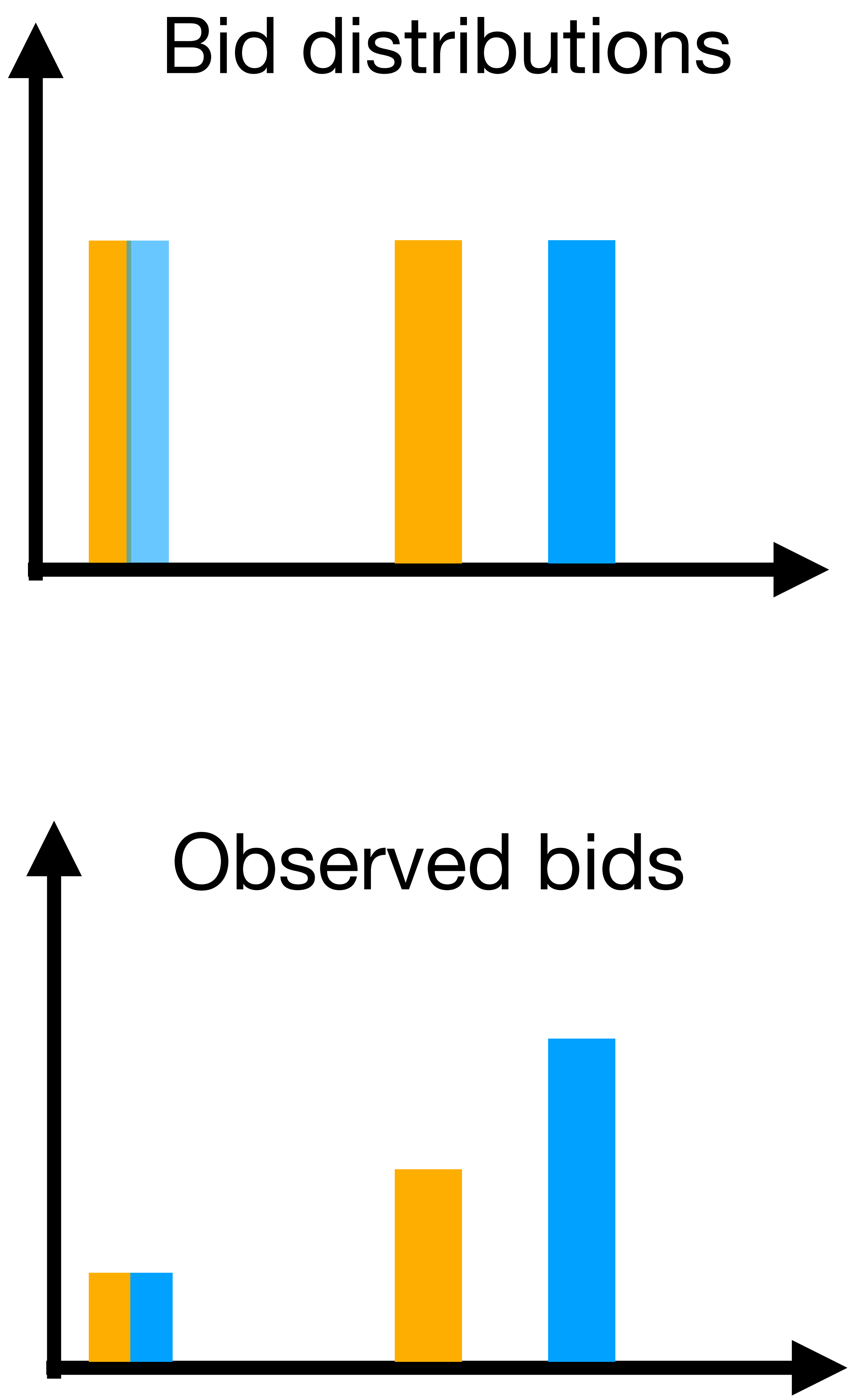

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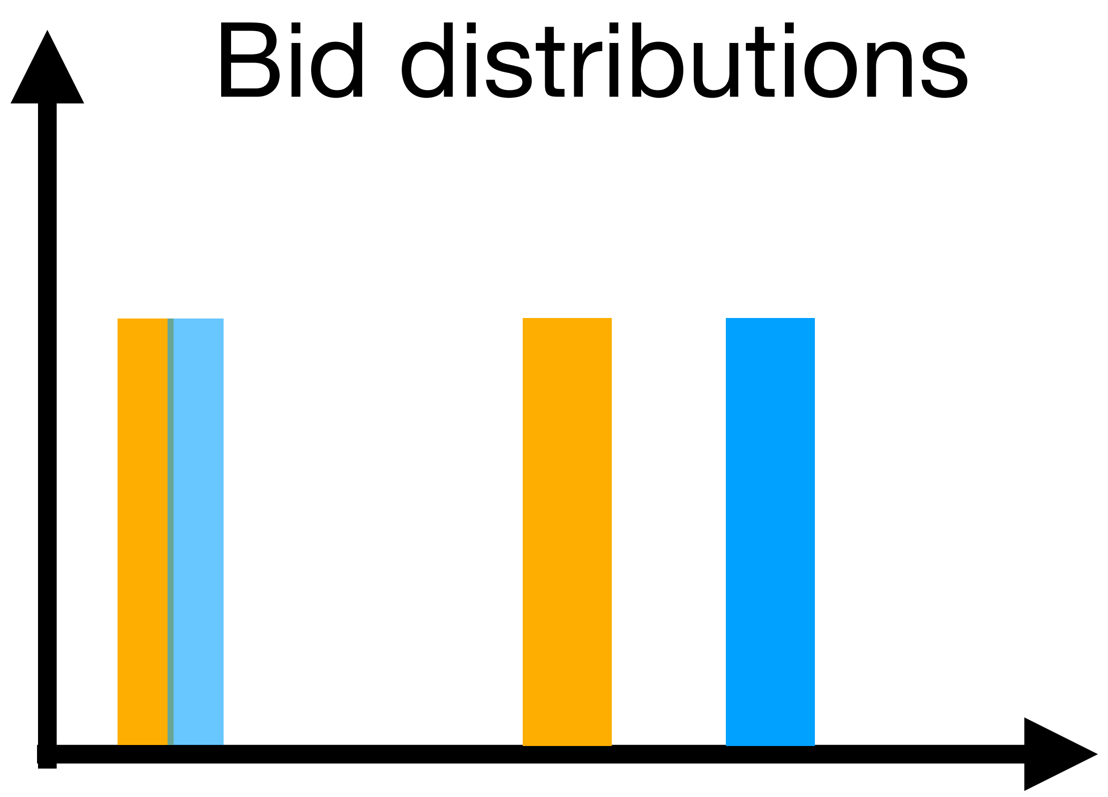

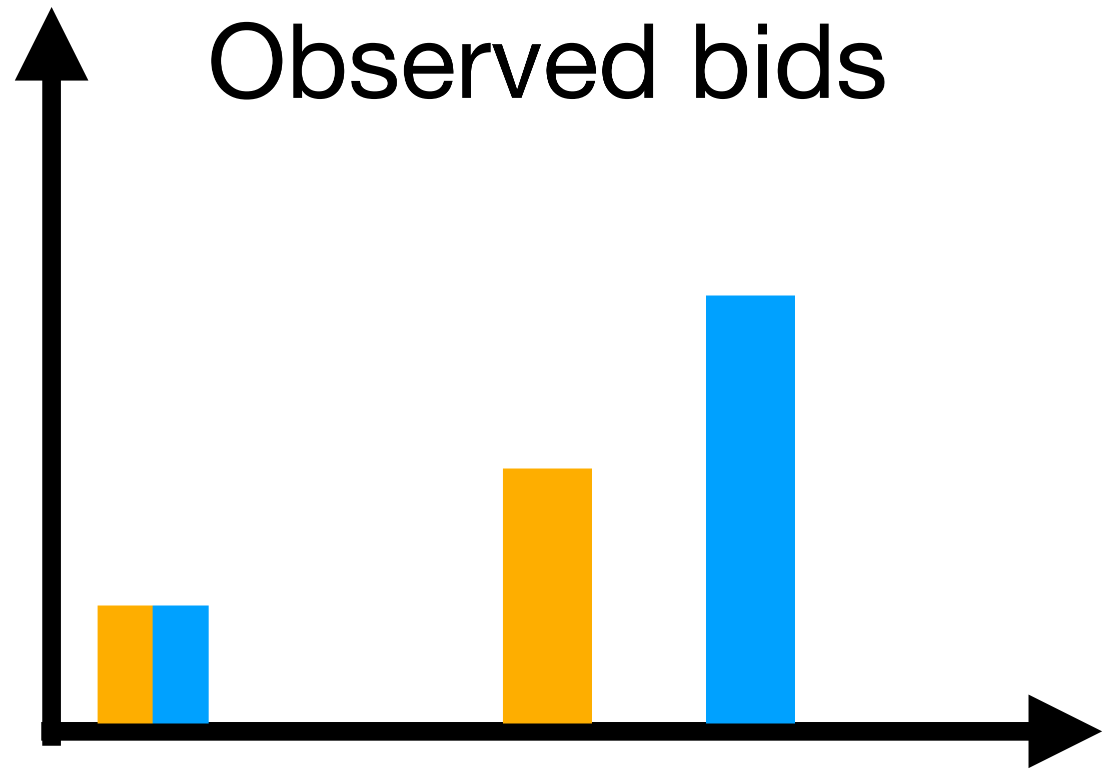

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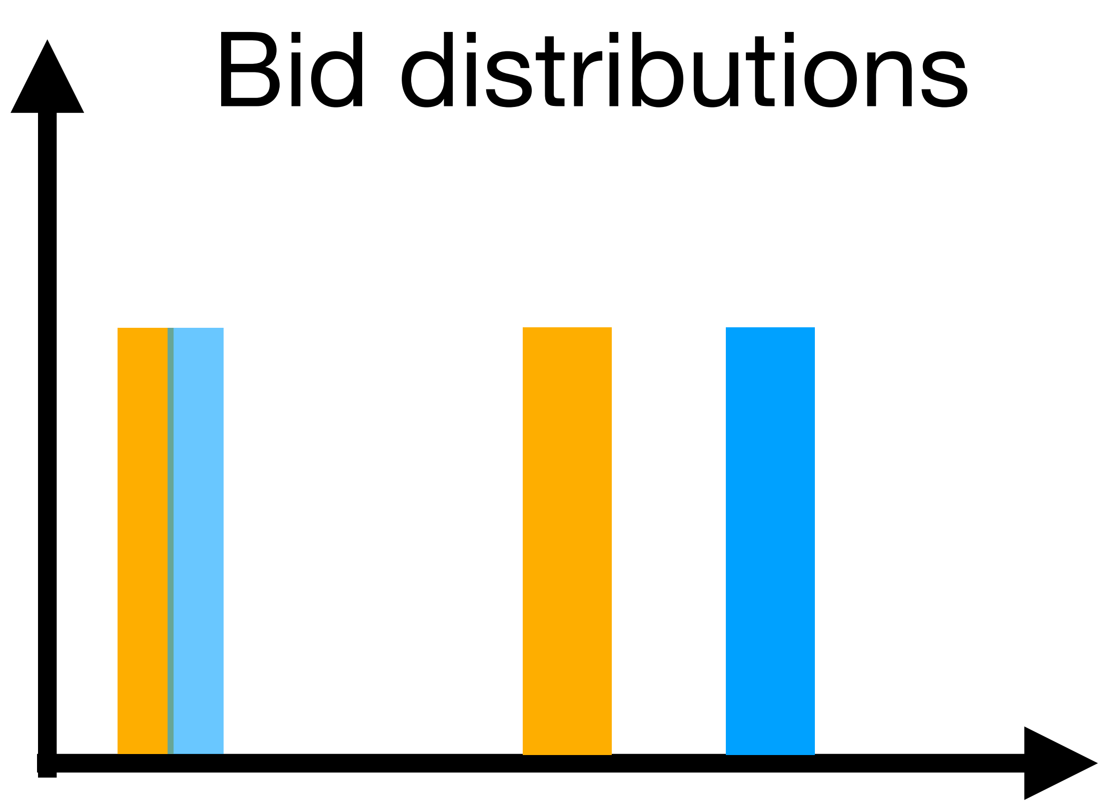
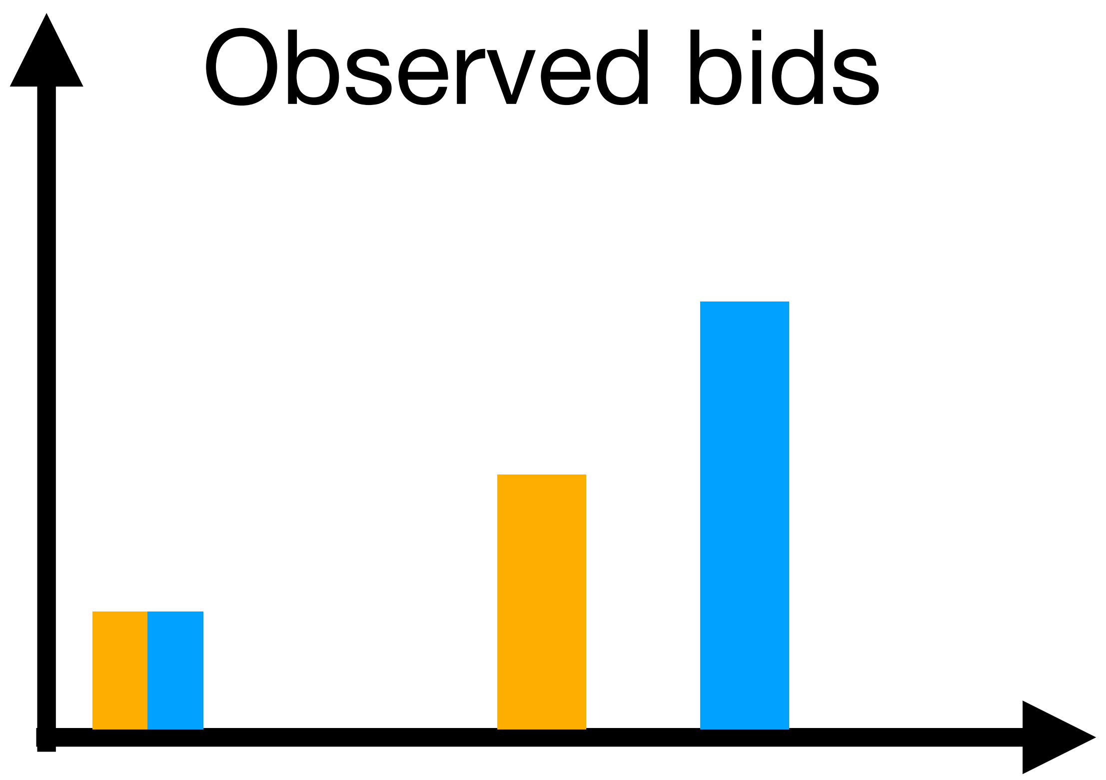
- **Goal:** Estimation with minimal assumptions (no Lipschitz densities, tail conditions, smoothness, etc.)





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- Partial observation model (Blum, Mansour & Morgenstern, 2015)

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Theorem: Can compute $\widehat{F_i}$ such that $\mathscr{W}(F_i, \widehat{F_i}) \leq \epsilon$

w.p. $1 - \delta$ using $O\left((\epsilon/2\lambda)^{4k} \cdot \log(1/\delta)\right)$ samples

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- **Solution:** learning over *effective support (Blum, Mansour & Morgenstern '15)*

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Theorem: The same algorithm yields $\sup_{x \in [p,1]} |F_i(x) - \widehat{F_i}(x)| \leq \epsilon$

w.p. $1 - \delta$ using $O\left(\log(k/\delta)/(\gamma^4 \epsilon^2)\right)$ samples

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  - Don’t even observe winning price, just the winner
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- We can adapt our algorithm to get $O(k \log(L))$ dependence in this setting

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Our result: Recover *value* distributions under BNE with extra $k^2/\text{poly}(\gamma)$ factor

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Theorem: With probability $1 - \delta$, we have $\sup_{x \in [0,1]} |F_i(x) - \widehat{F_i}(x)| \leq \epsilon$

for $\epsilon \leq e^{-Ck}$ using $O\left((1/\epsilon)^{Ck} \cdot \log(1/\delta)\right)$ samples

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Paper: https://arxiv.org/abs/2205.02060